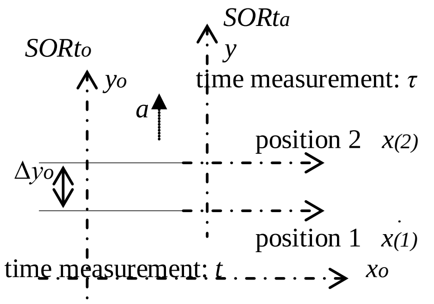
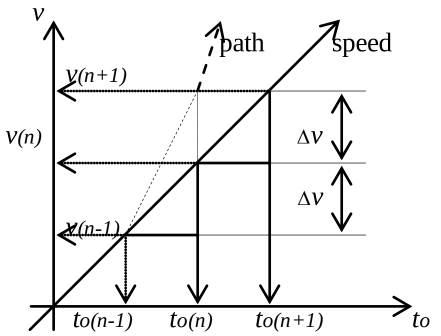
The Universal Work Formula

Acceleration *SORta* in *SORto*



*SOR****ta*** is accelerating with $a = const$ in direction *yo*. For a certain period of time it moves from *position 1* to *position 2* within *SOR****to***. Axis *y* is parallel with *yo*. Axis *xo* and *x* are also parallel with each other.
We divide the process of the acceleration into an infinite number of equal and *as small as possible* time periods.



The accelerating process has been divided into an infinite number of equal periods, with, for each period, constant speed.

The speed grows in time points $t_{o(n-1)}$, $t_{o(n)}$ and $t_{o(n+1)}$. It is constant in each period, but period by period of higher value, as follows:

$t_{o(n+1)} - t_{o(n)} = \Delta t_o$ and

for this period the speed is $v_{(n)} = const$

$t_{o(n)} - t_{o(n-1)} = \Delta t_o$ and

for this period the speed is $v_{(n-1)} = const$

and so on and $v_{(n-1)} < v_{(n)} < v_{(n+1)}$

This also means and identical to: the speed grows only for the balance of the mass change: $\dfrac{dv}{d\tau_a} = 0$ and $\dfrac{dv}{dt_o} = 0$

The time formula is: $d\tau = dt_o \dfrac{1}{\sqrt{1 - v^2/c^2}}$

The distance, *SOR****ta*** makes for the smallest possible time period within *SOR****to***, measured both the time and distance in *SOR****to*** is:

$$\frac{dy_o}{dt_o} = v \quad \text{and} \quad v = at_o \quad \text{and} \quad dy_o = at_o dt_o$$

The speed is reciprocal $\dfrac{dy}{d\tau} = v$ therefore:

$$dy = vdt_o \frac{1}{\sqrt{1 - \left(v^2/c^2\right)}};$$

$$dy = at_o \frac{1}{\sqrt{1 - \left(a^2 t_o^2/c^2\right)}} dt_o$$

The length of the path of the acceleration of *SOR****ta*** in direction *yo* from position 1 to 2 *measured* <u>within *SOR****ta***</u> is

$$\Delta y = s_a = \int at_o \frac{1}{\sqrt{1 - \left(a^2 t_o^2/c^2\right)}} dt_o$$

$$s_a = \frac{c^2}{a}\left(1 - \sqrt{1 - \frac{a^2 t_o^2}{c^2}}\right); \quad \textbf{(a)}$$

From **(a)** already is seen that the later deduced work formula for the count of the internal mass change is:

$$dW = s_a \cdot dF = s_a \cdot a \cdot dm =$$

$$\text{and} \quad W = mc^2\left(1 - \sqrt{1 - \frac{a^2 t_o^2}{c^2}}\right)$$

where $E = mc^2$ **obviously is the original mass-energy !**

What is the work, necessary to accelerate *SOR****ta***, a system of reference of mass *m* within *SOR****to*** <u>for the count</u> of its “mass-energy” from speed *0* to *v* with constant *a*?

The work values, measured within *SOR****ta*** and *SOR****to*** respectively are:

$$dW_a = \frac{dp_a}{d\tau_a} ds_a \quad \text{and} \quad dW_o = \frac{dp_o}{dt_o} ds_o$$

which give:

$$dW_a = \frac{dp_a}{d\tau_a} ds_a = \frac{d(mv)}{d\tau_a} ds_a = \left(\frac{dm}{d\tau_a} v + \frac{dv}{d\tau_a} m\right) ds_a$$

and

$$dW_o = \frac{dp_o}{dt_o} ds_o = \frac{d(mv)}{dt_o} ds_o = \left(\frac{dm}{dt_o} v + \frac{dv}{dt_o} m\right) ds_o$$

The drive of the acceleration is the internal energy of the mass; the growth of the speed is proportional to $dv = adt_o$; the value of the acceleration, measured in *SOR**to*** is $a = const$. The change of the work values within the two systems of reference are:

$$dW_a = \frac{dm}{d\tau_a} vds_a \, ; \quad \text{and} \quad dW_o = \frac{dm}{dt_o} vds_o \quad \text{accordingly}$$

The description of the length of the path of the acceleration at a certain time moment *to* is:

$$ds_a = ds_o \frac{1}{\sqrt{1-\left(a^2t_o^2/c^2\right)}}$$

and the time relation at this time moment is:

$$d\tau_a = dt_o \frac{1}{\sqrt{1-\left(a^2t_o^2/c^2\right)}}$$

The speed values, result of acceleration, measured within *SOR**to*** and *SOR**ta*** are reciprocal!
Therefore:

$$dW_a = \frac{dm}{d\tau_a} vds_o \frac{1}{\sqrt{1-\left(a^2t_o^2/c^2\right)}} \, ; \quad \text{gives} \quad d(dW_a) = \frac{dm}{d\tau_a} adt_o ds_o \frac{1}{\sqrt{1-\left(a^2t_o^2/c^2\right)}}$$

$$dW_o = \frac{dm}{d\tau_a\sqrt{1-\left(a^2t_o^2/c^2\right)}} vds_o \, ; \quad \text{gives} \quad d(dW_o) = \frac{dm}{d\tau_a\sqrt{1-\left(a^2t_o^2/c^2\right)}} adt_o ds_o$$

Work values are obviously equal, but the components within the formula are different:

- The *accelerating force*, measured within *SOR**ta*** is:

$$F_a = \frac{dp_a}{d\tau_a} = v\frac{dm}{d\tau_a} \quad \text{gives} \quad dF_a = adt_o \frac{dm}{d\tau_a} \quad \text{and} \quad dF_a = a\sqrt{1-\left(a^2t_o^2/c^2\right)}dm$$

- The *accelerating force*, measured within *SOR**to*** is:

$$F_o = \frac{dp_o}{dt_o} = v\frac{dm}{\sqrt{1-\left(a^2t_o^2/c^2\right)}d\tau_a} \quad \text{gives} \quad F_o = \frac{adt_o dm}{\sqrt{1-\left(a^2t_o^2/c^2\right)}d\tau_a} \quad \text{and} \quad dF_o = adm$$

$$\text{Consequently} \quad F_a < F_o \quad \text{and} \quad F_o = F_a \frac{1}{\sqrt{1-\left(a^2t_o^2/c^2\right)}}$$

- less F_a force acts for longer period $d\tau_a$ and in longer path ds_a.
- larger value of F_o force acts for less dt_o time period and in less ds_o path.

Whereas the speed relation between *SOR**ta*** and *SOR**to*** is reciprocal, the measured values of the *acceleration*, from the point of view of the systems of reference are different:

$$a_{(SORta)} = \frac{dv}{d\tau_a} = \frac{dv}{dt_o}\sqrt{1-\frac{a^2t_o^2}{c^2}} \, ; \quad a_{(SORta)} \neq a_{(SORto)}$$

For coming to correct results, we need to use *mass values*, which include this difference, the impulse of the change (acceleration).

$dF_a = \frac{dv}{d\tau_a}dm = \frac{dv}{dt_o}dm\sqrt{1-\frac{a^2t_o^2}{c^2}} = \frac{dv}{dt_o}dm_a$ $impulse = \frac{dF_a}{dv} = \frac{dm}{d\tau_a}$	The *measured intensity* of the mass change (the drive of the motion) within the two systems of reference are significantly different $\frac{dm_a}{dt} = -\frac{dm}{dt}\sqrt{1-\frac{a^2t_o^2}{c^2}}$
$dF_o = \frac{dv}{dt_o}dm \, ; \quad impulse = \frac{dF_o}{dv} = \frac{dm}{dt_o}$	

With substitutions

$dt_o = d\tau_a \sqrt{1-(a^2 t_o^2 / c^2)}$;

$dm_a = dm\sqrt{1-(a^2 t_o^2 / c^2)}$

$dW = dW_a = dW_o$

as $ds_o = v dt_o = a t_o dt_o$

$\frac{dW}{dm_a} = C - c^2 \cos\alpha$

taking $C = c^2$

The solution of the work formulas for the both systems of reference above is:

$$d(dW) = \frac{dm_a}{\sqrt{1-\left(a^2 t_o^2 / c^2\right)}} \frac{\sqrt{1-\left(a^2 t_o^2 / c^2\right)}}{dt_o} a \frac{dt_o ds_o}{\sqrt{1-\left(a^2 t_o^2 / c^2\right)}}$$

and $d(dW) = \frac{dm_a}{\sqrt{1-\left(a^2 t_o^2 / c^2\right)}} a ds_o = \frac{dm_a}{\sqrt{1-\left(a^2 t_o^2 / c^2\right)}} a^2 t dt_o$

$$d\frac{dW}{dm_a} = a^2 \int \frac{t_o}{\sqrt{1-\left(a^2 t_o^2 / c^2\right)}} dt_o$$

The solution of the integral is: $\frac{dW}{dm_a} = c^2\left(1-\sqrt{1-\frac{a^2 t_o^2}{c^2}}\right)$

The result is:

$$\boxed{W = W_o = W_a = m_a c^2\left(1-\sqrt{1-\frac{a^2 t_o^2}{c^2}}\right)}$$

which in the case of $dt_o = d\tau_a$ gives the *Newtonian* formula: $W_a = W_o = \frac{1}{2} m_a a^2 t_o^2$

The absolute work values for the two systems of reference are equal. They however correspond to different intensities for time period $\Delta\tau_a = \tau_a - 0 = \tau_a$ and time period $\Delta t_o = t_o - 0 = t_o$ respectively:

$w_a = \frac{dW_a}{d\tau_a} = \frac{dm_a c^2}{d\tau_a} - \frac{dm_a c^2}{d\tau_a}\sqrt{1-\frac{v^2}{c^2}}$	means: *acceleration for the count* ***of the internal energy of the mass***
$w_o = \frac{dW_o}{dt_o} = \frac{dm_a c^2}{dt_o} - \frac{dm_a c^2}{dt_o}\sqrt{1-\frac{v^2}{c^2}} = \frac{dm_a c^2}{d\tau_a\sqrt{1-\frac{v^2}{c^2}}} - \frac{dm_a c^2}{d\tau_a}$ ↑ *increase of the effect of the mass change-* - result of the ***intensification*** of the internal mass change process	means: *acceleration for the count* ***of external energy*** *– different than the internal energy of the mass*

The formulas above demonstrate the similarity and the difference in the approaches: the institutional one and the one introduced at this site.